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BLOWING-UP COORDINATES FOR A SIMILARITY BOUNDARY LAYER EQUATION

BERNARD BRIGHI AND TEWFIK SARI

Université de Haute Alsace, Laboratoire de mathématiques
F.S.T., 4 rue des frères Lumière, 68093 MULHOUSE (France)

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Abstract. We introduce blowing-up coordinates to study the autonomous third order nonlinear differential equation : $f''' + \frac{\mathbf{m}+1}{2} ff'' - \mathbf{m}f'^2 = 0$ on $(0, \infty)$, subject to the boundary conditions $f(0) = a \in \mathbb{R}$, $f'(0) = 1$ and $f'(t) \to 0$ as $t \to \infty$. This problem arises when looking for similarity solutions to problems of boundary-layer theory in some contexts of fluids mechanics, as free convection in porous medium or flow adjacent to a stretching wall. We study the corresponding plane dynamical systems and apply the results obtained to the original boundary value problem, in order to solve questions for which direct approach fails.

1. **Introduction.** We consider the autonomous third order nonlinear differential equation

$$f''' + \frac{\mathbf{m}+1}{2} ff'' - \mathbf{m}f'^2 = 0 \quad \text{on} \quad (0, \infty), \tag{1.1}$$

subject to the boundary conditions

$$f(0) = a, \tag{1.2}$$

$$f'(0) = 1, \tag{1.3}$$

$$f'(\infty) := \lim_{t\to\infty} f'(t) = 0. \tag{1.4}$$

The parameters $\mathbf{m}$ and a will be assumed to describe $\mathbb{R}$, and we are concerned by existence and uniqueness questions for solutions of problem (1.1)-(1.4). In the case $\mathbf{m} = 0$, equation (1.1) reduces to the so-called Blasius equation, and has been widely studied (see [6], [12], [17], [19], [22] and [23]). This boundary value problem arises when looking for similarity solutions in physically different contexts in fluids mechanics, as free convection about a vertical flat surface embedded in a fluid-saturated porous medium (see [10], [11], [14], [18]), or boundary-layer flow adjacent to a stretching wall (see [3], [4], [13], [16], [20]). The parameter $\mathbf{m}$ is related to some conditions given on the wall, while a corresponds, for example for the stretching wall, to an impermeable wall when $a = 0$, to a permeable wall when $a \neq 0$, say suction ($a > 0$) or injection ($a < 0$) of the fluid. In these physical papers problem (1.1)-(1.4) is essentially studied from numerical point of view, or by using formal expansions, and only some elementary results are proved. Further mathematical

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